# FINE TUNING GPS CLOCK ESTIMATION IN THE MCS

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**Abstract**

***With the completion of a 24 operational satellite constellation, GPS is fast approaching the critical milestone, Full Operational Capability (FOC). Although GPS is well capable of providing the timing accuracy and stability figures required by system specifications, the GPS community will continue to strive for further improvements in performance.***

***The GPS Master Control Station (MCS) recently demonstrated that timing improvements are always possible, provided we don't sacrifice system integrity in the process. The most recent improvements have concentrated on a re-evaluation of the MCS Kalman Filter's Continuous Time Update Process Noises, also known as qs. Rubidium (Rb) qs received notable (and well needed) attention in early 1994. In late 1994, the MCS completely re-assessed the qs for all individual GPS frequency standards.***

***By tuning MCS clock estimation on a satellite-by-satellite basis, we've safely optimized the utility of the GPS Composite Clock, and hence, Kalman Filter state estimation, providing a small improvement to user accuracy.***

## INTRODUCTION

Though well capable of meeting and/or exceeding customer expectations, the GPS Master Control Station (MCS) will continuously search for safe and efficient methods for improving GPS timing accuracy and stability performance. The most recent improvements have focused on fine tuning the Continuous Time Update Process Noises (a.k.a. $q$s) for all GPS satellite frequency standards.

Process noises are nothing new to the timing community. Many time scale algorithms update these parameters dynamically for their respective systems. As in many Kalman Filters, the Defense Mapping Agency (DMA) periodically reviews their $q$ values for their OMNIS computation program. OMNIS, like the MCS Kalman Filter, estimates the ephemeris, solar, and clock states for 25 GPS satellites [3]. However, up until 6 Oct 94, the timing community had never undertaken the task of re-$q$ing an entire operational GPS constellation in the *MCS* Kalman Filter.

Thanks to the generous input from several outside agencies, we now employ process noise values that are unique to the individual characteristics of the 25 operating frequency standards on orbit. Perhaps more importantly, we now also have the precise data, know-how, tools, and procedures to safely and efficiently review and update our $q$ values on a periodic basis.

# RUBIDIUM CLOCK ESTIMATION

Each GPS satellite uses one of two different types of atomic clocks to provide a stable output frequency, to, in turn, generate accurate navigation signals. The majority of Block II/IIA GPS satellites currently use one of two available Cesium (Cs) frequency standards. Orbiting Cs clocks demonstrate reliable performance, with one-day stabilities ranging between 0.8 E-13 to 2.0 E-13 [13,14,15]. The drift rate term for a Cs frequency standard is typically on the order of 1 E-20 $s/s^2$ or less. Such a small drift rate term, an order of magnitude smaller than our time steering magnitude, has negligible effects on GPS timing (hundredths of a nanosecond over one day). Because of its relatively insignificant effect on frequency estimation, the MCS currently fixes the drift rate estimate to zero for all Cs frequency standards (on-orbit and ground based).

Two Rubidium (Rb) clocks also reside on each Block II/IIA satellite. Rb clocks *do* exhibit a significant aging characteristic, typically on the order of 1 E-18 $s/s^2$. However, if a Filter properly corrects for drift rate, the typical one-day frequency stability of a Rb clock *state* is significantly better than that of a Cs (0.6 E-13 versus 1.0 E-13) [13,14,15]. Unfortunately, in the past, our Kalman Filter had difficulty estimating drift rate. As a result, Rb clock estimates have had somewhat large variances, causing, in turn, increased difficulty in estimating frequency. Although a Rb clock itself is usually more stable than a Cs at one day, the stabilities of the MCS's Kalman Filter Rb clock *states* have, in the past, been *worse* than those for Cs clocks.

This Filter instability has impeded the MCS from incorporating their inherently better stability into GPS time calculations. Consequently, the timing community has been uneasy about using Rb clocks in GPS. Of the first 24 operational satellites, we initialized only *three* with Rubidium clocks.

Despite this reluctant attitude towards using Rubidium clocks, many have realized that as Cesium clocks reach their respective ends of operational life, we will have *no choice* but to use more Rubidium clocks. In any case, it seemed counterintuitive that GPS was not making the most use of our most stable clocks. In early 1994, the 2 SOPS Navigation Analysis Section began tackling this long-standing concern. Because the problem resided in estimation, as opposed to physical clock performance, the Kalman Filter really only needed a fine tuning.

## Deriving New Rubidium Clock *qs*

The MCS Kalman Filter performs recursive time and measurement updates of the state residuals and covariances. In pure prediction, the clock state covariances are functions of the system *qs* [18]:

$$P = \begin{bmatrix} q_1 t + q_2 t^3/3 + q_3 t^5/20 & q_2 t^2/2 + q_3 t^4/8 & q_3 t^3/6 \\ q_2 t^2/2 + q_3 t^4/8 & q_2 t + q_3 t^3/3 & q_3 t^2/2 \\ q_3 t^3/6 & q_3 t^2/2 & q_3 t \end{bmatrix} \quad (1)$$

The Naval Research Laboratory (NRL) produced a report for 2 SOPS (ALL-5, 27 Jan 94), on SVN25. The report included a series of drift rate plots for the Rb clock that was active from Mar 1992 until Dec 1993. NRL plotted 5, 10, 20, and 30-day averaged values for drift rate [11]. In analyzing the 30-day average plot, we noticed that the drift rate changed significantly more during the first 90 days than during the *remaining operational time [figure 1]*:

From the above P matrix, in pure Filter prediction, the system variance for drift rate is the scalar time product of $q_3$:

$$C_3 = q_3(\tau) \qquad (2)$$

Using the above equation, along with the NRL data, we derived new $q$ values, both from the 90-day initialization period, and from the remaining period, and we compared these to the old system $q$ values:

| $q$ Value | OLD | INITIAL | NEW NORMAL |
|---|---|---|---|
| **Drift Rate ($q_3$)** | 9.00 E-42 $s^2/s^5$ | 1.35 E-43 $s^2/s^5$ | 6.66 E-45 $s^2/s^5$ |

We also looked at calculating a new drift (frequency) $q$ value. The old Rb $q$ value for drift, 4.44 E-32 $s^2/s^3$, was the same as that for Cs. We chose $q_2$ = 2.22 E-32 $s^2/s^3$ [4]. Again, to be conservative, and to allow the Filter to handle any possible instability resulting from clock "warm-up", we set the initialization $q_2$ value to 3.33 E-32 $s^2/s^3$. We kept the phase (bias) $q$ unchanged. Below is a comparison of the old set and the two new sets of process noise values for Rubidiums:

| $q$ Value | OLD | INITIAL | NEW NORMAL |
|---|---|---|---|
| **Bias ($q_1$)** | 1.11 E-22 $s^2/s$ | 1.11 E-22 $s^2/s$ | 1.11 E-22 $s^2/s$ |
| **Drift ($q_2$)** | 4.44 E-32 $s^2/s^3$ | 3.33 E-32 $s^2/s^3$ | 2.22 E-32 $s^2/s^3$ |
| **Drift Rate ($q_3$)** | 9.00 E-42 $s^2/s^5$ | 1.35 E-43 $s^2/s^5$ | 6.66 E-45 $s^2/s^5$ |

Of course, one might question using 30-day averaged drift rate values for deriving $q_3$--could the drift rate change by an unacceptable amount during those 30 days, thus undermining the premise of these calculations? Well, in the past, NRL has been able to apply as much as a 150 day flat-average aging correction to their Allan Deviation plots--plots showing one-day stability figures similar to SVN25's [10]. The implication is, if the Filter has a good drift rate term, that value can essentially be fixed for, in some cases, up to 150 days, without significantly degrading the one-day accuracy of the other clock states. Certainly, assuming drift rate consistency over 30 days, let alone 150, was safe for deriving the above $q$ values for the MCS Kalman Filter.

## SVN9 End Of Life Testing

50th Space Wing approved a 1 SOPS and 2 SOPS joint effort to conduct End Of Life testing on SVN9 during March and April 1994 [19]. As part of the plan, Rockwell suggested dedicating 7-8 days for testing Rubidium clock drift rate estimation. We used the "New Normal" $q$ values, and monitored the resulting system performance.

The test, which lasted 8.7 days, produced very encouraging results [19]. At the end of the test, with tighter process noise values, the Kalman Filter converged on a drift rate value of -2.38 E-18 $s/s^2$, with an associated standard deviation of 1.99 E-19 $s/s^2$ (compared to a typical standard deviation of 1.0 E-18 $s/s^2$, using the old $q$ values). Using an off-line tool, Rockwell derived post-processed values for comparison. Using a simple slope of their $A_1$ (frequency) estimates over 7 days, Rockwell's drift rate estimate was -2.44 E-18 $s/s^2$, well within one sigma of the Filter's estimate. The National Institute of Standards and Technology (NIST) Report on SVN9 End of Life Testing pointed to a value of -2.32 E-18 $s/s^2$ [8], also well within one sigma of the Filter estimate. These comparisons indicated that the Filter had performed as

designed--to converge on a more accurate drift rate estimate, with a correspondingly representative error estimate (standard deviation) [figure 2].

By, in effect, "clamping" on the Filter estimate, one must question whether this covariance tightening is too restrictive, limiting the Filter's capability to respond to normal clock movement. We used two MCS parameters to test this capability.

a. The first parameter was the Measurement Residual Statistical Consistency Test (MRSCT). Essentially, the MRSCT decides whether or not to accept Pseudoranges (PRs). Over 8.7 days, the Filter accepted each and every smoothed PR for SVN9. The average PR residual (PRR) was no higher than that of a typical healthy, operational vehicle, or SVN9's prior to the test.

b. The second parameter was the Estimated Range Deviation (ERD). The ERD gives a good indication of the range error a user is experiencing, based on the current navigation upload residing in the vehicle. Over the 8.7 days, we uploaded SVN9 only once per day, and the ERD RMS never once exceeded 3.1 meters--well within our ERD criteria of 10 meters. Correspondingly, the one-day User Range Accuracy (URA) dropped from 5.0 to 3.8 meters, and the four-day URA dropped from 33.0 to 13.0 meters. In hindsight, we could have even set SVN9 healthy during the test, and netted a small improvement to global coverage and accuracy [figure 2].

In short, results from the SVN9 drift rate test indicated that Filter estimation worked quite better with the reduced process noise ($q$) values.

## Real World Implementation Of The New Rubidium $q$s

On 18 Mar 94, we began applying these results towards real-world SVN10 and SVN24 clock estimation. Since, at that time, SVN24's Rb was less than three months old, and since SVN10 is a Block I, always susceptible to the effects of eclipse seasons, we selected the "Initialization" $q$s instead of the "New Normal" $q$s.

For SVN10, during the three months prior to the test, ERDs exceeded 5.0 meters on 19 separate days. During the three months after the new $q$s were installed, SVN10 ERDs didn't once exceed 4.8 meters. In addition, our Smoothed Measurement Residual (SMRES) tool showed that SVN10 residuals from the DMA monitor stations, since 18 Mar 94, have been consistent with those prior to 18 Mar 94, as well as those for our other satellites. Similarly, between these two time periods, SVN10's time transfer error dropped from 14.6 to 9.9 nanoseconds (RMS), according to United States Naval Observatory (USNO) data [5]. These data points, from independent agencies, further show a significant improvement in satellite accuracy.

Similar to SVN10's, the ERDs for SVN24 decreased after 18 Mar 94. Additionally, after installing the "New Normal" $q$s on 24 Apr 94, from that time to the present, the Filter has easily and consistently accepted SVN24 PRs. Likewise, SVN24 residuals from DMA, since 24 Apr 94, have been as good or better than those prior to 24 Apr 94, and better than those of the other 23 operational satellites. In terms of upload accuracy, SVN24's ERDs routinely exceeded 4.0 meters prior to 24 Apr 94. Since 24 Apr 94, SVN24's ERDs have rarely exceeded 3.5 meters, and have typically stayed under 2.5 meters. SVN24, now, is one of our two most accurate satellites. To complete the usefulness of this improvement, on 28 Apr 94, we included SVN24 into the GPS composite clock, allowing it to better stabilize GPS time.

For the time being, after the 2 SOPS has initialized a Rubidium clock for 7-14 days, we'll probably install the "Initialization" *q*s for 90 days. At the three month point, assuming nominal clock performance, we'll likely install the "New Normal" *q*s. Also, at three months, we will aggressively consider including that satellite into the GPS composite clock--a Block II/IIA Rubidium clock estimate, now properly corrected for drift rate, now has a *better* one-day frequency stability than those of each of the on-orbit Cesiums. The GPS community, as a whole, can now at least tame a long existing ambivalence we've had about using Rubidium clocks in operational satellites. A Rubidium clock, now properly tuned in the Kalman Filter, significantly improves GPS timing and positioning accuracies. Currently, five GPS satellites use Rubidium clocks. One, in particular, SVN36, is arguably now our *most accurate* satellite.

# CESIUM CLOCK ESTIMATION

Having resolved perhaps the most significant recent problem with GPS clock estimation through improved Rubidium *q*s, we decided to expand this opportunity for improvement to the remainder of all on-orbit GPS frequency standards: Cesium (Cs) clocks. As demonstrated earlier, deriving clock *q*s involves two main steps: 1) obtaining data that can accurately describe the behavior of the clocks involved, and 2) mathematically translating this behavior into the *q*s themselves.

DMA has already been doing exactly this. A snapshot of some recently-derived DMA *q*s shows values that are, for the most part, unique to the individual clocks [3]. DMA's *q*s vary significantly between satellites. In contrast, prior to 6 Oct 94, the MCS *q*s were *equal* for most GPS Cs clocks. Also noteworthy is that the MCS's $q_1$ value was less than *each* of DMA's equivalent $q_1$ values [3]:

| | **MCS *q* Values** | **DMA *q* Values** | |
|---|---|---|---|
| | | **Smallest** | **Largest** |
| **Bias ($q_1$)** | 1.11 E-22 $s^2/s$ | 4.25 E-22 $s^2/s$ | 4.35 E-21 $s^2/s$ |
| **Drift ($q_2$)** | 4.44 E-32 $s^2/s^3$ | 7.35 E-33 $s^2/s^3$ | 5.16 E-32 $s^2/s^3$ |

This comparison raised two questions: 1) Would uniquely tuning the *q*s provide a significant improvement to GPS performance? 2) Does a legitimate reason exist for deliberately having lower $q_1$ terms in the MCS Kalman Filter? The remainder of this paper answers the first question. The second question, however, is more philosophical.

MCS software experts will argue that a fundamental difference in purpose between the respective Kalman Filters at the MCS and at DMA constitutes a legitimate reason for using different $q_1$ terms. Since the MCS Kalman Filter is designed, in part, to provide accurate 24 hour predictions for navigation uploads, one could argue that we might want to deliberately keep our $q_1$ low to reduce the gain, and hence, prevent a situation whereby a noisy Kalman update could skew a 24-hour navigation upload prediction. Timing experts, however, will argue that tinkering with this parameter can be dangerous, since doing so can impose a configuration inconsistent with the basic intended design of a Kalman Filter. Both sides have very legitimate arguments.

## Deriving New Cesium Clock *q*s

Analysts at NRL provide timely, accurate, and understandable reports on GPS clock performance. In particular, we now greatly utilize their Allan Deviation [$\sigma(\tau)$] plots, created from DMA precise ephemeris

data. The following equation relates the Allan Variance [$\sigma^2(\tau)$] to Kalman Filter *q*s. This equation assumes independence between each sample frequency pair [2].

$$\sigma^2(\tau) = q_1(\tau^{-1}) + q_2(\tau)/3 + q_3(\tau^3)/20 \quad (3)$$

In order to relate current clock performance (via the Allan Deviation) to the system *q*s, we try not to use data more than 90 days old. Unfortunately, by only using 90 days of data, we experience the tradeoff of degraded confidence intervals for $\tau > 20$ days. For Cesium clocks, this is a non-concern, since we currently fix the drift rate and $q_3$ values to zero. For Rubidium clocks, however, the degraded confidence intervals, combined with the difficulty of correcting for drift rate without violating the sample frequency pair independence assumption, makes calculating the last term dangerous. As demonstrated earlier in this paper, we now have very suitable $q_3$ values for Rubidium clocks. Thus, for $\tau < 20$ days, we can substitute these into the Allan Variance equation, and simply solve for $q_1$ and $q_2$. Then, we can compare our theoretical values to empirical values, using NRL Allan Deviation plots (with flat aging corrections applied for Rubidium clocks) [12].

One other concern relates to measurement noise. The data from NRL, and hence from DMA, has a fairly certain amount of measurement noise. The MCS's parameter for measurement noise, which we'll call $q_0$, accounts for some of the GPS monitor station (MS) receiver noise, some of the satellite clock's white and flicker phase noise, MS location errors, and general modeling errors. DMA has a similar parameter designed to account for measurement noise, currently set to $(45 \text{ cm})^2 \cong 0.20 \text{ m}^2$. For years, the MCS set this parameter at $1.0 \text{ m}^2$. Thanks to recently refined MS location coordinates from DMA [6], the MCS was recently able to reduce $q_0$ to $(0.86\text{m})^2 \cong 0.74 \text{ m}^2$. We derived this value using 500 Pseudorange Residual values from a widely distributed assortment of times and satellite-MS combinations. Our new value of $0.74 \text{ m}^2$, not surprisingly, is not a dramatic reduction from $1.0 \text{ m}^2$, but nonetheless is consistent with our expectation of improvement from the new coordinates:

$$\sqrt{(1.00^2 - 0.86^2)} = .51 \text{ (meters)} \quad (4)$$

One might suggest using DMA's lower value. However, since our parameter accounts for more than just pure white measurement "noise", our parameter is higher for a legitimate reason. Although not purely white phase noise in nature, noise associated with measurements can tend to misrepresent the stability of the estimated clock states. We can roughly express the instability resulting from this representation error as [1]:

$$\sigma_r^2(\tau) = 3q_0(\tau^{-2}) \quad (5)$$

By assuming independence between this representation error and the other noise processes on a given clock, the equation for the Allan Variance of the measured clock adds an additional term [7]:

$$\sigma_m^2(\tau) = 3q_0(\tau^{-2}) + q_1(\tau^{-1}) + q_2(\tau)/3 + q_3(\tau^3)/20 \quad (6)$$

We created a Basic program to plot the theoretical $\sigma(\tau)$ values, using the above equation, for $\tau = 0.1$ to 100 days. Using recent precise ephemeris $\sigma(\tau)$ plots from NRL [12], along with the Basic program, we derived new *q* values for all satellites [figure 3]. Note that the Rubidium *q*s remained *unchanged*. The Rb *q*s we derived earlier this year are, and have been, consistent with true clock performance. Nonetheless, Figure 4 shows how the theoretical Allan Deviation *does* change significantly for, in particular, SVN21 and SVN23, by using the newer *q*s.

The current MS bias and drift $q$s, 1.11 E-22 $s^2/s$ and 4.44 E-32 $s^2/s^3$, respectively, are *not* representative of true MS clock performance. However, the MCS uses three separate mini-Kalman Filters, a.k.a. "partitions" to individually estimate MS clock states. Since a ***partition reconciliation*** algorithm keeps these states fairly consistent [1,4], over time, the MCS estimation structure effectively triples the weighting of the long term effects of MS clocks. With this current $q_2$ value for MSs, this "triple weighting" produces, in a roundabout fashion, the *effect* of using a $q_2$ roughly the same as the smallest satellite $q_2$. We may tweak this parameter in the future, but, for the time being, this effect produces a fairly accurate result [16].

We also began using a newer set of $q$s during Cesium clock initialization. Below is a comparison of the old $q$s, and new initialization $q$s we've derived:

| Old $q_1$ E-22 $s^2/s$ | Old $q_2$ E-32 $s^2/s^3$ | Old $q_3$ E-45 $s^2/s^5$ | New $q_1$ E-22 $s^2/s$ | New $q_2$ E-32 $s^2/s^3$ | New $q_3$ E-45 $s^2/s^5$ |
|---|---|---|---|---|---|
| 1.11 | 4.44 | 0 | 4.44 | 3.33 | 0 |

## Testing The New Cesium $q$ Values

We safely tested the validity of these changes on 3 Oct 94, using a Test & Training simulator in the MCS. The results were impressive.

a. As expected, the state covariances converged to steady state values more truly representative of the unique short- and long-term variances of the individual clocks. Also as expected, none of these new steady state covariances differed drastically from the typical older values. The implication of these small, but significant changes is that the Filter safely re-weighted clock state estimation based on true frequency standard performance, as opposed to assumed performance equality (equal $q$s):

| Value | OLD VARIANCES (All Cs) | NEW VARIANCES (Minimum) | (Maximum) |
|---|---|---|---|
| **Bias** | 1.25 E-17 $s^2$ | 1.07 E-17 $s^2$ | 1.83 E-17 $s^2$ |
| **Drift** | 3.20 E-27 $s^2/s^2$ | 1.38 E-27 $s^2/s^2$ | 4.38 E-27 $s^2/s^2$ |

b. As expected, the current state residuals experienced small (not trivial, not severe) changes, indicating that the Filter more responsibly distributed error to the appropriate states.

c. The MCS Pseudorange Residuals (PRRs) dropped from 1.61 m (RMS) to 0.87 m (RMS), after the Filter reprocessed the same raw data with the new set of $q$s. This more dramatically indicates that the Filter more responsibly distributed error to the appropriate states, so well that Filter predictions can now have less systematic error, and hence, less error when compared to smoothed measurements.

d. The consistency of MS clock states across the Kalman Filter partitions experienced a small, but not trivial improvement (A 3.8 % reduction in Bias divergence error, and 21.6 % reduction in Drift divergence error). Again, by more responsibly appropriating error to the respective clock states, short-term MS clock state instability across the partitions dropped.

## Real-World Implementation Of The New Cesium *q*s

By installing these new *q*s on 6 Oct 94, we safely improved a) Kalman Filter clock estimation, b) navigation error representation, and c) the stability of the GPS composite clock.

The stability of GPS time, defined by the GPS composite clock, intuitively, should have improved simply as a result of the improved weighting, again, by uniquely tuning the *q*s based on true clock performance. When we used equal *q*s, the Allan Variance, $\sigma^2_\Delta(\tau)$, of the implicit ensemble of N equally weighted clocks (for $\tau$ = 1 day) was approximately [4]:

$$\sigma^2_\Delta(\tau) \cong 1/(N^2) \sum_{i=1}^{N} \sigma^2_{yi}(\tau) \qquad (7)$$

Using the one-day Allan Deviation figures from NRL Quarterly Report 94-3 [15], the one-day stability of this implicit ensemble was approximately **1.55 E-14**.

By using clock-unique *q*s, the Allan Deviation of the now finely tuned implicit ensemble (for $\tau$ = 1 day) is approximately [1,4]:

$$\sigma^2_\Delta(\tau) \cong [\sum_{i=1}^{N} (\sigma^2_{yi}(\tau))^{-1}]^{-1} \qquad (8)$$

Incorporating the same one-day NRL Allan Deviation figures into the above equation, the one-day stability of the implicit ensemble dropped to approximately **1.22 E-14**. Similarly, the *observed* Allan Deviation of GPS time, derived from USNO-smoothed measurements [5,9], also dropped, not only for $\tau$ = 1 day, but for $1 \leq \tau \leq 10$ days [figure 5].

Important to note is a large improvement in extended (14 day) navigation performance. By utilizing more representative (lower) $q_2$ values, the 14-day URA predictions have dropped to lower, more representative values for most satellites. Figure 6 shows a comparison of the typical 14-day URA values before and after 6 Oct 94, for all Block IIA satellites in estimating partitions. Though not an absolute indication of extended navigation accuracy, by uniquely tuning the *q*s, these URA values now, at least, have more validity than before. The 14-day URA values for all healthy GPS satellites, since 6 Oct 94, have been well below the NAVSTAR GPS System Operational Requirements Document (SORD) User Range Error (URE) specification of 200 meters [17].

## CONCLUSION

This fine tuning reinforces how deriving and installing clock-unique MCS Kalman Filter process noise values can safely and significantly improve GPS timing performance. We will continue to update these parameters on a regular basis. In the near future, we plan to review these values every three months, and as needed (after a clock swap or a dramatic change in clock performance).

Loral Federal Systems Division received a tasking to more comprehensively review these and other data base parameters in 1995. We expect the results from their analysis to be more precise than the above results, due to the extensive background of the team of experts that will tackle this project.

Nonetheless, this successful attempt at fine tuning the MCS *qs* helps pave a path for future MCS data base analyses, and hence for future refinements to GPS timing performance.

## ACKNOWLEDGMENTS

The author wishes to thank the following people and agencies for their generous assistance with both our timing improvements and this paper:

Ronald Beard, NRL
Don Brown, Major, USAF Reserve
Ken Brown, Loral Federal Systems Division
Jim Buisson, SFA
James W. Chaffee, Associate Member, IEEE
M. K. Chien, Loral Federal Systems Division
Bruce Hermann, Naval Surface Warfare Center
Steve Holm, Retired, Defense Mapping Agency
Judah Levine, NIST
Steve Malys, Defense Mapping Agency
Sam R. Stein, Timing Solutions Corporation
Everett R. Swift, Naval Surface Warfare Center
John V. Taylor IV, formerly of Rockwell International, now with Martin Marietta
The people of the 2 SOPS
Francine Vannicola, USNO
Marc A. Weiss, NIST

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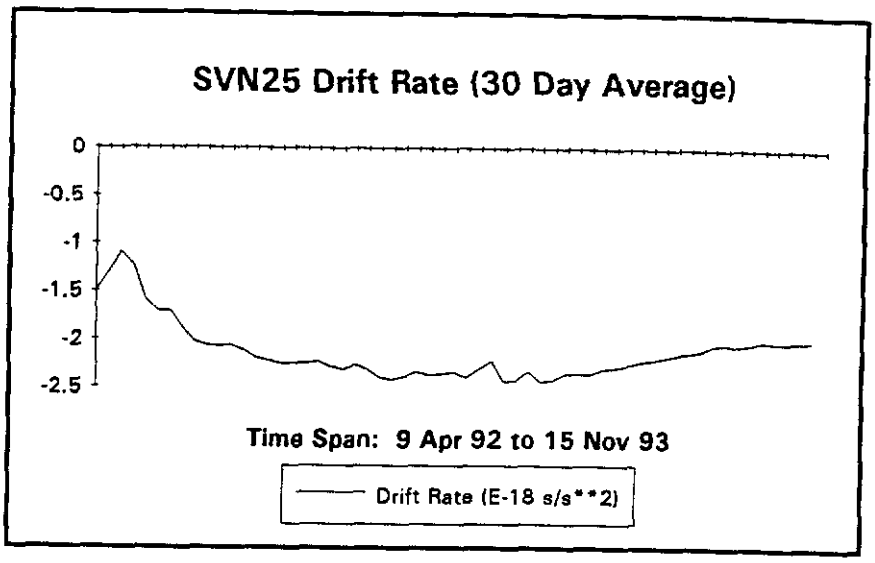
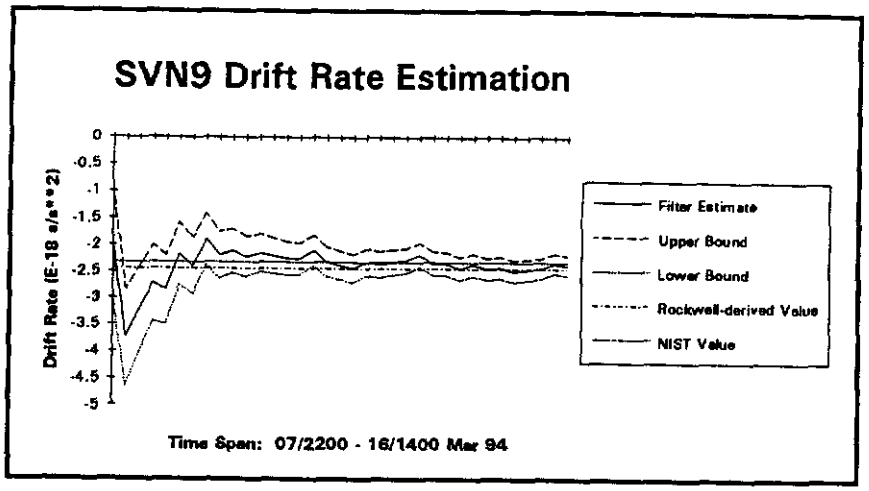
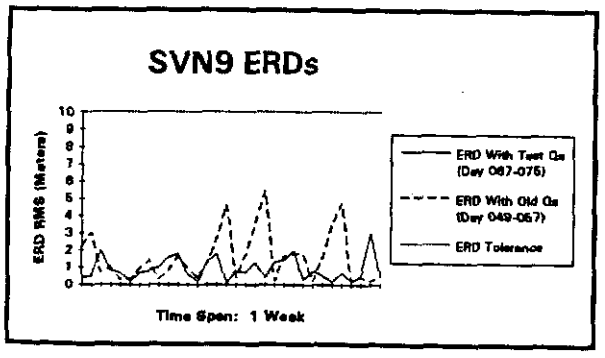



**Figure 1**



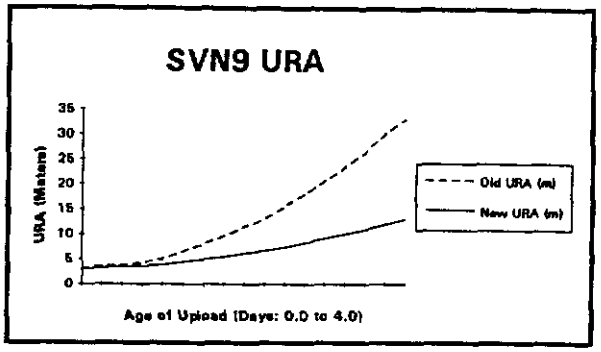
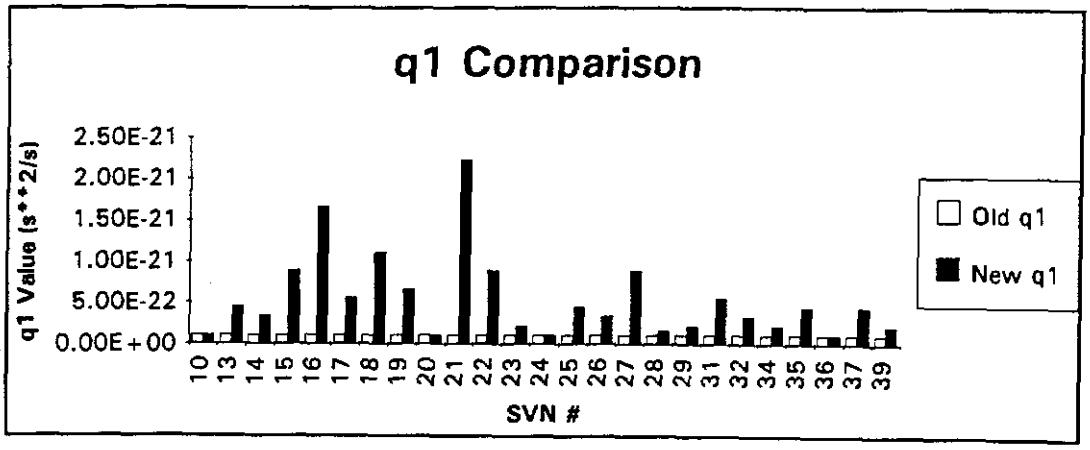




**Figure 2**



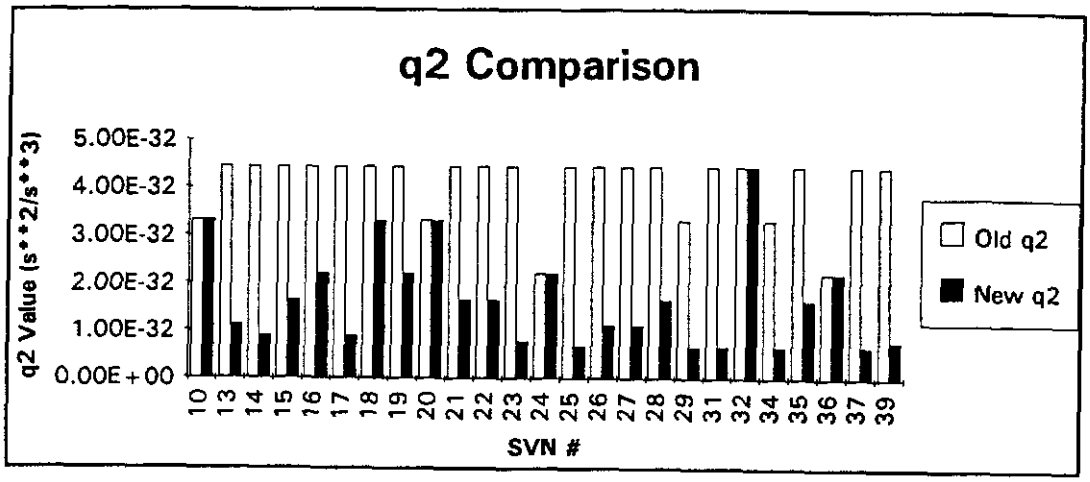
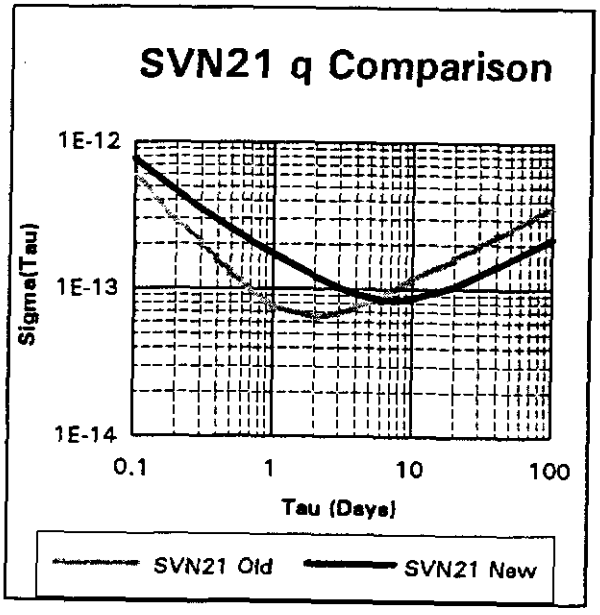




**Figure 3**



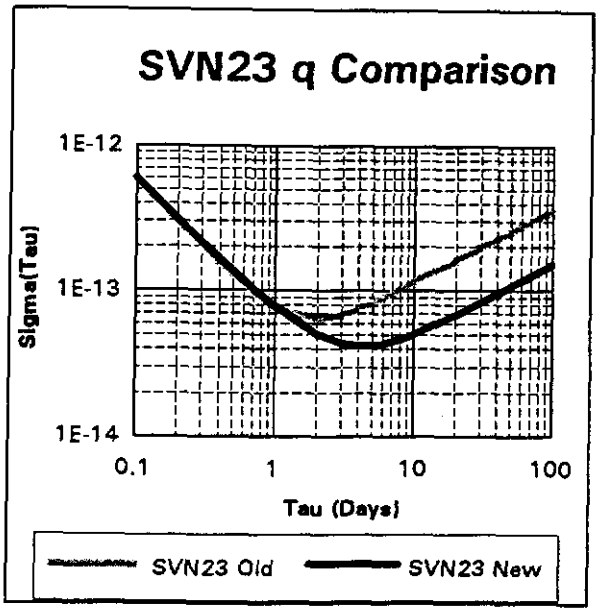




**Figure 4**

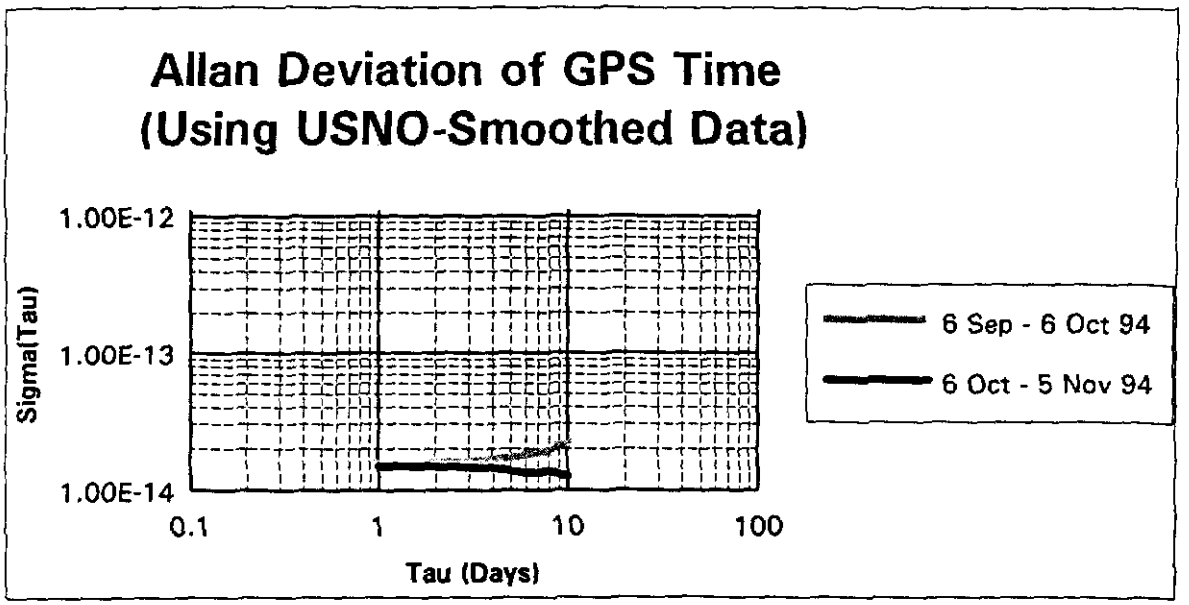



Figure 5

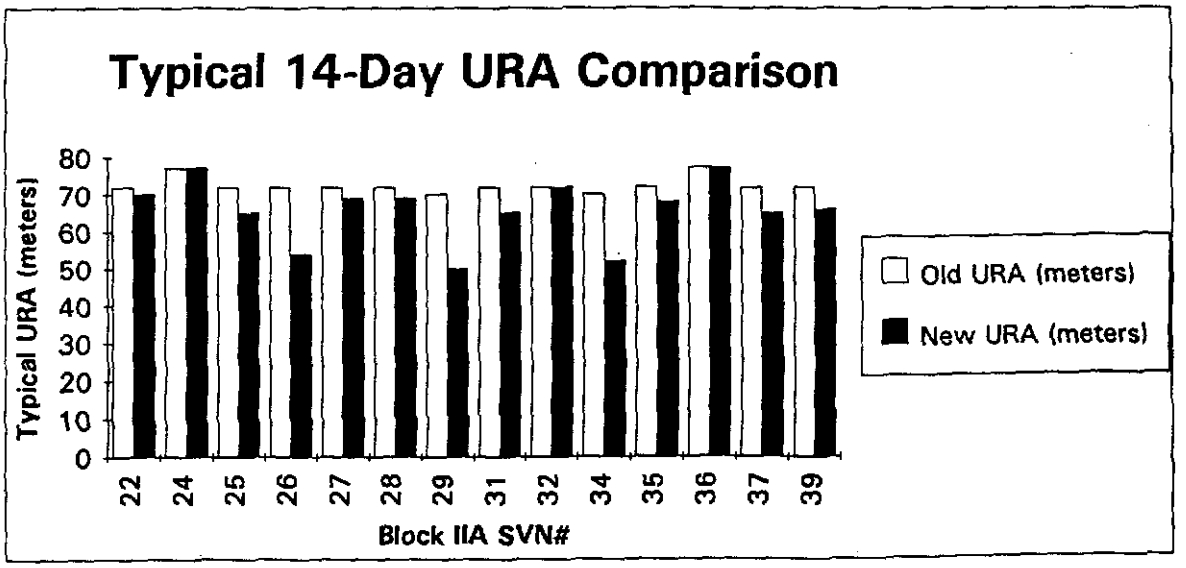



Figure 6